

Area law of the entropy in the critical gravity

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Abstract

The entropy of the Schwarzschild-anti de Sitter black hole in the recently proposed four-dimensional critical gravity is trivial in the Euclidean action formulation, while it is expressed by the area law in terms of the brick wall method given by 't Hooft. To resolve this issue, we relate the Euclidean action formulation to the brick wall method semiclassically, and show that the entropy of the black hole can be expressed by the area law at the critical point.

Keywords: Euclidean Path Integral, Brick Wall, Black Hole, Thermodynamics

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1 Introduction

Since it has been claimed that the general relativity is nonrenormalizable, there have been extensive studies for quantum theory of gravity such as string theory, conventional perturbative gravity, and so on. In particular, one of the perturbatively renormalizable gravity theories can be built by adding quadratic curvature terms to the Einstein gravity [1, 2]. However, theories including higher-order time-derivative terms should endure massive ghost modes. In recent studies on the three-dimensional topologically massive gravity [3, 4] including a cosmological constant, it has been shown that there exists some critical point such that the massive mode becomes massless and carries no energy, so that the problem can be solved [5].

Similarly, in the four-dimensional quadratic gravity theory with a cosmological constant, one can find a critical point, where the massive ghost mode disappears. This model is called the *critical gravity* [6] defined by

$$I_{\text{CG}}[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2\Lambda + \alpha \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) \right] \quad (1)$$

with a cosmological constant Λ . At the critical point, $\alpha = 3/2\Lambda$, in spite of the renormalizability, it seems to be trivial in the sense that the mass and the entropy of a Schwarzschild-anti de Sitter (SAdS) black hole which is a solution to this theory become zeroes [6]. Moreover, this result can be also confirmed by the Euclidean action formulation of the black hole thermodynamics [7, 8, 9].

On the other hand, it has been suggested by Bekenstein that the intrinsic entropy of a black hole is proportional to the surface area at the event horizon [10, 11, 12], and then quantum field theoretic calculation has been given for the Schwarzschild black hole by Hawking [13]. Actually, one of the best way to reproduce the area law of black holes is to use the brick wall method suggested by 't Hooft [14]. By considering fluctuation of matter field around black holes semiclassically, one can always get the desired results, however, this result is not compatible with the result of the Euclidean action formulation for the critical gravity.

In this paper, we would like to resolve the above-mentioned issue and study how to derive the entropy satisfying the area law in terms of the Euclidean action formulation. First task is to get the nontrivial free energy by taking into account higher-order corrections in the Euclidean path integral and then the corresponding entropy can be nontrivial. For convenience, the fluctuation of the metric field will be ignored, i.e. our calculations will be performed in semiclassical approximations. We recapitulate the Euclidean action formulation by carefully considering the appropriate boundary term in section 2. In contrast to conventional cases, the entropy is trivially zero assuming the critical condition. It means that the partition function is trivial so that the area law of the entropy does not appear. So, we consider the one loop correction of the scalar degrees of freedom around the black hole in section 3 and relate the Euclidean action formulation to the brick wall method semiclassically. Eventually, the free energy turns out to be nontrivial even at the critical condition where it is actually compatible with the free energy from the brick wall method with some conditions, which yields the area law of the black hole entropy in section 4. Finally, in section 5, summary and some discussions are given.

2 Thermodynamics with Euclidean action formulation

We start with a minimally coupled scalar field ϕ coupled to the critical gravity as $I_{\text{tot}} = I_{\text{CG}} + I_{\phi}$, where the scalar field action is

$$I_{\phi}[g, \phi] = - \int d^4x \sqrt{-g} \left[\frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 \right]. \quad (2)$$

For $\phi = 0$, the SAdS black hole is just a classical solution to this model. The line element of the SAdS black hole is given by $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2^2$ with

$$f(r) = 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2 = \left(1 - \frac{r_h}{r} \right) \left[1 - \frac{\Lambda}{3} (r^2 + r_h r + r_h^2) \right], \quad (3)$$

where $M = (r_h/2G) (1 - \Lambda r_h^2/3) > 0$ is the mass parameter of the black hole, $\Lambda < 0$ is the cosmological constant, and r_h is the radius of the horizon. The free energy $F^{(0)}$ for

this vanishing scalar solution can be obtained from Euclidean action formulation [7, 8, 9],

$$Z^{(0)}[g] = \exp(iI_{\text{CG}}[g]) = \exp(-\beta F^{(0)}) . \quad (4)$$

The crucial ingredient for this calculation is to find the consistent boundary term. Following ref. [15], an auxiliary field $f_{\mu\nu}$ is introduced to localize the higher curvature terms so that the Euclidean version of the action (1) and the corresponding boundary term can be written in the form of

$$I_{\text{CG}} = -\frac{1}{16\pi G} \int_{\mathcal{M}} d^4x \sqrt{g} \left[R - 2\Lambda + f^{\mu\nu} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) - \frac{1}{4\alpha} f^{\mu\nu} (f_{\mu\nu} - g_{\mu\nu} f) \right], \quad (5)$$

$$I_{\text{B}} = -\frac{1}{16\pi G} \int_{\partial\mathcal{M}} d^3x \sqrt{\gamma} \left[2K + \hat{f}^{ij} (K_{ij} - \gamma_{ij} K) \right], \quad (6)$$

where γ_{ij} and K_{ij} are the induced metric and the extrinsic curvature of the boundary, respectively. And \hat{f}^{ij} in the boundary term is defined as $\hat{f}^{ij} = f^{ij} + f^{ri} N^j + f^{rj} N^i + f^{rr} N^i N^j$ with $N^i = -g^{ri}/g^{rr}$ for the hypersurface described by $r = r_0$. In the Euclidean geometry, the Euclidean time is defined by $\tau = it$ and should be identified by $\tau = \tau + \beta_H$ to avoid a conical singularity at the event horizon, where β_H is the inverse of the Hawking temperature.

Next, taking the boundary to the infinity, $r_0 \rightarrow \infty$, the free energy is obtained as

$$F^{(0)} = \beta_H^{-1} (I - I_{\text{vacuum}}) = [1 - 2\alpha\Lambda/3] \frac{r_h}{4G} \left(1 + \frac{\Lambda}{3} r_h^2 \right), \quad (7)$$

where $I = I_{\text{CG}} + I_{\text{B}}$ and $I_{\text{vacuum}} = I|_{M=0}$, and the Hawking temperature for the given metric function (3) is calculated as

$$T_H = \beta_H^{-1} = \frac{1 - \Lambda r_h^2}{4\pi r_h}. \quad (8)$$

Then, the thermodynamic first law reads the entropy and the energy of the black hole,

$$S^{(0)} = \beta_H^2 \frac{\partial F^{(0)}}{\partial \beta_H} = [1 - 2\alpha\Lambda/3] \frac{\pi r_h^2}{G}, \quad (9)$$

$$E^{(0)} = F^{(0)} + \beta_H^{-1} S^{(0)} = [1 - 2\alpha\Lambda/3] \frac{r_h}{2G} \left(1 - \frac{\Lambda}{3} r_h^2 \right), \quad (10)$$

which are exactly same with those obtained in ref. [6]. Note that the factor $[1 - 2\alpha\Lambda/3]$ is vanishing at the critical point $\alpha = 3/2\Lambda$. Thus, we can confirm that the energy and the

entropy of the SAdS black hole at the critical point are vanishing also in the Euclidean action formulation.

As was mentioned in the previous section, the entropy from the brick wall method satisfies the area law and gives the nontrivial thermodynamic quantities such as energy and heat capacity. At first glance, there seems to exist incompatibility between the Euclidean action formulation and the brick wall method. In what follows, it will be shown that the semiclassical treatment of the Euclidean action formulation can be related to the brick wall method if we introduce the cutoff.

3 Semiclassical Euclidean action formulation

Now, we take the classical background as the SAdS black hole metric along with $\phi = 0$, and then consider the fluctuated quantum field semiclassically. The partition function up to one loop order for the scalar field is expressed as

$$\begin{aligned} Z[g] &= Z^{(0)}[g]Z^{(1)}[g] \\ &= \exp(-\beta F^{(0)}) \exp(-\beta F^{(1)}) \\ &= e^{iI_{\text{CG}}[g]} \int \mathcal{D}\phi e^{iI_\phi[g, \phi]}, \end{aligned} \tag{11}$$

where the total free energy consists of $F = F^{(0)} + F^{(1)}$. Note that the tree level free energy $F^{(0)}$ is trivial at the critical point as seen in the previous section, so that the nontrivial contribution to the free energy should come from the one loop effective action.

The one loop partition function $Z^{(1)}$ can be written as

$$\begin{aligned} Z^{(1)}[g] &= \int \mathcal{D}\phi e^{iI_\phi} \\ &= \det^{-1/2}(-\square + m^2), \end{aligned} \tag{12}$$

and the effective action W_ϕ becomes

$$\begin{aligned} W_\phi &= \frac{i}{2} \ln \det(-\square + m^2) \\ &= \frac{i}{2} \text{Tr} \ln(-\square + m^2) \end{aligned}$$

$$= \frac{i}{2} \int \frac{d^4x d^4k}{(2\pi)^4} \ln(k_\mu k^\mu + m^2), \quad (13)$$

where k_μ corresponds to the conjugate momentum to x^μ . Note that a (covariant) Fourier transform in curved spacetimes has not been established [16, 17, 18, 19]. However, the manifold can be split into a number of small pieces, in which we can consider a Riemann normal coordinates, i.e. $\int_{\mathcal{M}} d^4x \sqrt{-g} \simeq \sum_{U \subset \mathcal{M}} \int_U d^4\tilde{x}$, where \tilde{x} represents the Riemann normal coordinates [20]. Then, one can perform the calculation in the momentum space by using the Fourier transform defined in this coordinates, $-\Box + m^2 \rightarrow \tilde{k}_\mu \tilde{k}^\mu + m^2$, where \tilde{k} is the momentum measured in the local coordinates. Consequently, it is possible to recover the global coordinates back for the covariant result (13).

In the Euclidean geometry, the time component of the four vector k_μ becomes $2\pi n/(-i\beta)$, and the integrals $\int dt$ and $\int dk_0/(2\pi)$ can be replaced by $-i \int d\tau$ and $(-i\beta)^{-1} \sum_n$, respectively [21]. Then, the Euclidean one loop effective action at the finite temperature is written as

$$\begin{aligned} W_\phi &= \frac{1}{2} \sum_n \int \frac{d^3x d^3k}{(2\pi)^3} \ln \left(\frac{4\pi^2 n^2}{f\beta^2} + E_m^2 \right), \\ &= \beta \int \frac{d^3x d^3k}{(2\pi)^3} \left[\frac{\sqrt{f} E_m}{2} + \frac{1}{\beta} \ln \left(1 - e^{-\beta \sqrt{f} E_m} \right) \right], \end{aligned} \quad (14)$$

where the relation $\sum_n \ln \left(\frac{4\pi^2 n^2}{\beta} + E_m^2 \right) = 2\beta \left[\frac{E_m}{2} + \frac{1}{\beta} \ln \left(1 - e^{-\beta E_m} \right) \right]$ was used and E_m is defined by

$$E_m^2 \equiv g^{ij} k_i k_j + m^2 = f k_r^2 + \frac{k_\theta^2}{r^2} + \frac{k_\phi^2}{r^2 \sin^2 \theta} + m^2. \quad (15)$$

Actually, the first term in the Euclidean one loop effective action (14) is related to the vacuum energy of the spacetime which is independent of the black hole temperature and we define the temperature dependent free energy as $\beta F^{(1)} = W_\phi - W_\phi^{\text{vacuum}}$.

Now, introducing a new parameter ω , the free energy can be rewritten as

$$F^{(1)} = \frac{1}{\beta} \int d\omega \int \frac{d^3x d^3k}{(2\pi)^3} \delta(\omega - \sqrt{f} E_m) \ln(1 - e^{-\beta \omega}). \quad (16)$$

Since the delta function can be written as the derivative of a step function, $\delta(x) = \frac{d}{dx} \epsilon(x)$, defining the step function as $\epsilon(x) = 1$ for $x > 0$ and 0 for $x < 0$, then the free energy (16)

can be calculated as

$$F^{(1)} = \frac{1}{\beta} \int \frac{d^3x d^3k}{(2\pi)^3} \epsilon(\omega - \sqrt{f} E_m) \ln(1 - e^{-\beta\omega}) \Big|_{-\infty}^{\infty} - \int d^3x \int_{m\sqrt{f}}^{\infty} d\omega \frac{1}{e^{\beta\omega} - 1} \int_{V_p} \frac{d^3k}{(2\pi)^3} \quad (17)$$

by performing the integration by parts. V_p is the volume of the phase space satisfying $\sqrt{f} E_m \leq \omega$, which can be explicitly written as

$$f k_r^2 + \frac{k_\theta^2}{r^2} + \frac{k_\phi^2}{r^2 \sin^2 \theta} \leq \frac{\omega^2}{f} - m^2, \quad (18)$$

for given values of ω and m . Subsequently, the first term in eq. (17) is vanishing, the free energy can be given by

$$F^{(1)} = - \int d^3x \int_{m\sqrt{f}}^{\infty} d\omega \frac{1}{e^{\beta\omega} - 1} \int_{V_p} \frac{d^3k}{(2\pi)^3} = - \int_0^{\infty} d\omega \frac{1}{e^{\beta\omega} - 1} \int_{V_p} \frac{d^3x d^3k}{(2\pi)^3}, \quad (19)$$

where the relation $\int d^3x \int_{m\sqrt{f}}^{\infty} d\omega = \int_0^{\infty} d\omega \int_{\omega > m\sqrt{f}} d^3x$ was used.

Finally, the integration with respect to x and k can be replaced by the number of quantum states with energy less than ω ,

$$n(\omega) \equiv \int_{V_p} \frac{d^3x d^3k}{(2\pi)^3} = \frac{1}{(2\pi)^3} \int_{V_p} dr d\theta d\phi dk_r dk_\theta dk_\phi, \quad (20)$$

then the free energy (19) for a scalar field can be written as the remarkably familiar form of

$$F^{(1)} = - \int d\omega \frac{n(\omega)}{e^{\beta\omega} - 1}. \quad (21)$$

It is interesting to note that the free energy (21) obtained from the finite temperature one loop effective action for a scalar field is the same with that given by the brick wall method suggested by 't Hooft [14] as it should be.

4 Entropy of a Schwarzschild-anti de Sitter black hole

In this section, we will calculate the thermodynamic quantities of the SAdS black hole using the free energy (21). First, the number of quantum states (20) with energy less

than ω for a spherically symmetric black hole is calculated as

$$n(\omega) = \frac{2}{3\pi} \int dr \frac{r^2}{\sqrt{f}} \left(\frac{\omega^2}{f} - m^2 \right)^{3/2}. \quad (22)$$

Then, the free energy (21) is obtained as

$$\begin{aligned} F^{(1)} &= -\frac{2}{3\pi} \int dr \frac{r^2}{f^2} \int_{m\sqrt{f}}^{\infty} d\omega \frac{(\omega^2 - m^2 f)^{3/2}}{e^{\beta\omega} - 1} \\ &= -\frac{2}{3\pi\beta^4} \int dr \frac{r^2}{f^2} \int_{z_0}^{\infty} \frac{dz}{e^z - 1} (z^2 - z_0^2)^{3/2}, \end{aligned} \quad (23)$$

where $z \equiv \beta\omega$ and $z_0 \equiv \beta m\sqrt{f}$. Subsequently, it is explicitly written as

$$F^{(1)} = -\frac{2\pi^3 r_h^4}{45\beta^4 \epsilon} (1 - \Lambda r_h^2)^{-2} \quad (24)$$

in the leading order where r_h , ϵ are the event horizon of the black hole and the UV cutoff parameter. The UV cutoff ϵ is assumed to be very small compared to the event horizon with a condition $m^2 \ll r_+ / [\epsilon\beta^2(1 - \Lambda r_+^2)]$.

Now, following 't Hooft [14], let us define the proper length for the UV cutoff parameter as

$$\bar{\epsilon} \equiv \int_{r_h}^{r_h + \epsilon} dr \sqrt{g_{rr}} \approx \frac{2\sqrt{r_h \epsilon}}{\sqrt{1 - \Lambda r_h^2}}, \quad (25)$$

which is independent of the parameters of the black hole. Then, the free energy (24) is rewritten as

$$F^{(1)} = -\frac{8\pi^3 r_h^5}{45\beta^4 \bar{\epsilon}^2} (1 - \Lambda r_h^2)^{-3}. \quad (26)$$

The thermodynamic first law and the definition of the free energy gives

$$S^{(1)} = \beta^2 \frac{\partial F^{(1)}}{\partial \beta} \Big|_{\beta=\beta_H} = \frac{32\pi^3 r_h^5}{45\beta_H^3 \bar{\epsilon}^2} (1 - \Lambda r_h^2)^{-3}, \quad (27)$$

and the energy is obtained as

$$E^{(1)} = F^{(1)} + \beta^{-1} S^{(1)} \Big|_{\beta=\beta_H} = \frac{8\pi^3 r_h^5}{15\beta_H^4 \bar{\epsilon}^2} (1 - \Lambda r_h^2)^{-3}. \quad (28)$$

Recovering dimensions and plugging the Hawking temperature (8) into the entropy (27), one can get

$$S^{(1)} = \frac{\ell_p^2}{90\pi \bar{\epsilon}^2} \frac{c^3 \mathcal{A}}{4G\hbar}, \quad (29)$$

where $\mathcal{A} = 4\pi r_h^2$ and $\ell_p = \sqrt{G\hbar/c^3}$ are the area of horizon and the Plank length, respectively. The entropy (29) agrees with the Bekenstein-Hawking entropy $S^{(1)} = c^3 \mathcal{A}/(4G\hbar)$ when the cutoff is chosen as $\bar{\epsilon} = \ell_p/\sqrt{90\pi}$, which is exactly same as in the case of the Schwarzschild black hole [14]. Then, the free energy (26) and the energy (28) are rewritten as

$$F^{(1)} = -\frac{c^4 r_h}{16G}(1 - \Lambda r_h^2) = -\frac{c^3 \mathcal{A}}{16G\hbar\beta_H}, \quad (30)$$

$$E^{(1)} = \frac{3c^4 r_h}{16G}(1 - \Lambda r_h^2) = \frac{3c^3 \mathcal{A}}{16G\hbar\beta_H}. \quad (31)$$

Next, the heat capacity is calculated as

$$\begin{aligned} C_V^{(1)} &\equiv T_H \frac{\partial S^{(1)}}{\partial T_H} = T_H \left(\frac{\partial S^{(1)}}{\partial r_h} \right) \left(\frac{\partial T_H}{\partial r_h} \right)^{-1} \\ &= -\frac{c^3 \mathcal{A}}{2G\hbar} \left(\frac{1 - \Lambda r_h^2}{1 + \Lambda r_h^2} \right), \end{aligned} \quad (32)$$

which is positive for $r_h > 1/\sqrt{|\Lambda|}$ and negative otherwise. It means that the SAdS black hole is stable for large black holes and unstable for small black holes, as is well-known. For $\Lambda = 0$, the heat capacity (32) is always negative, which coincides with the thermodynamic stability of the Schwarzschild black hole.

5 Discussion

In summary, the four-dimensional critical gravity seems to be trivial in that the entropy of the SAdS black hole is vanishing, which may be a somewhat impatient conclusion because the black hole temperature is not zero which means that the Hawking radiation exists. In the Euclidean action formulation at finite temperature, the total free energy consists of the tree free energy and the one loop corrected free energy $F = F^{(0)} + F^{(1)}$, which yields the total entropy $S = S^{(0)} + S^{(1)}$. In spite of the vanishing tree entropy $S^{(0)} = 0$, the total entropy gives the area law by taking into account the quantum fluctuation of the scalar field.

So far, we have assumed that both the scalar decoupling condition and the critical condition are valid since the metric field has been fixed. Now, one may wonder

whether these conditions are still met or not when one considers the one loop back reaction of the geometry because it may affect the thermodynamic quantities. For this purpose, the coefficients in front of higher curvature terms can be released as $I[g] = \frac{1}{16\pi G_B} \int d^4x \sqrt{-g} [R - 2\Lambda_B + \alpha_B R_{\mu\nu} R^{\mu\nu} + \beta_B R^2]$, where the parameters are bare couplings defined by, e.g., $\alpha_B = \alpha + \hbar\delta\alpha$. Of course, it recovers the critical gravity for the particular choice of $\alpha + 3\beta = 0$ and $\alpha - 3/2\Lambda = 0$ at the tree level. Now, the one loop effective action of the massive scalar field whose mass is m can be also written in the form of the divergent higher curvature terms so that the renormalization yields $\alpha_R/G_R = \alpha_B/G_B + \hbar A/120\pi$ and $\beta_R/G_R = \beta_B/G_B + \hbar A/240\pi$, where α_R , β_R and G_R are the renormalized couplings. Note that $A \approx \ln(\mu^2/m^2) + 2\ln(2/3) + O(m^2/\mu^2)$ is a divergent constant for $\mu \rightarrow \infty$ [20, 23]. It means that $\alpha_R + 3\beta_R = (\hbar G_N/24\pi) \ln(2/3)$ and $\alpha_R - 3/2\Lambda_R = (\hbar G_N/480\pi\Lambda^2)[4(2\Lambda^2 + 60\Lambda m^2 - 45m^4) \ln(2/3) + 15m^2(8\Lambda - 9m^2)]$ by appropriate counter terms. Fortunately, by rescaling $\mu \rightarrow 3\mu/2$, one can still require $\alpha_R + 3\beta_R = 0$ in order to avoid the existence of the scalar graviton; however, the critical condition is still violated as $\alpha_R - 3/2\Lambda_R = (\hbar G_N/32\pi\Lambda^2)m^2(8\Lambda - 9m^2)$. Therefore, one cannot maintain the scalar graviton decoupling condition and the critical condition simultaneously on account of the back reaction of the geometry. But, further study is needed for the concrete argument and relevant thermodynamic quantities in connection with the back reaction of the geometry. We hope this issue will be addressed elsewhere.

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